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Quadratic Actions in Dependent Fields and the Action Principle

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Abstract General field theories are considered, within the functional *differential* formalism of quantum field theory, with interaction Lagrangian densities $\mathcal{L}_I(x;\lambda)$, with λ a generic coupling constant, such that the following expression $\partial\mathcal{L}_I(x;\lambda)/\partial\lambda$ may be expressed as quadratic functions in *dependent* fields but may, in general, be arbitrary functions of *independent* fields. These necessarily include, as special cases, present renormalizable gauge theories. It is shown, in a unified manner, that the vacuum-to-vacuum transition amplitude (the generating functional) may be explicitly derived in functional differential form which, in general, leads to modifications to computational rules by including such factors as Faddeev–Popov ones and *modifications* thereof which are explicitly obtained. The derivation is given in the *presence* of external sources and does not rely on any symmetry and invariance arguments as is often done in gauge theories and no appeal is made to path integrals.

Keywords Functional differential formalism of quantum field theory · Dependent fields · Action principle · Quantization rules · Gauge theories

1 Introduction

The purpose of this communication is to investigate systematically, in a unified manner, within the functional *differential* formalism of quantum field theory [1–13], field theories with interaction Lagrangian densities $\mathcal{L}_I(x;\lambda)$, with λ a generic coupling constant, such that $\partial\mathcal{L}_I(x;\lambda)/\partial\lambda$ may be expressed as quadratic functions in dependent fields and, in general, as arbitrary functions of independent fields. These include, as special cases, present renormalizable gauge field theories. For example, the non-abelian ones, such as QCD, are quadratic, while QED is linear in dependent fields. The functional differential treatment necessitates the introduction of *external sources* in order to generate the vacuum-to-vacuum transition amplitude, as a generating functional, from which amplitudes for basic processes

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may be extracted. The novelty of this work is that we show that for all the general Lagrangians, mentioned above, the vacuum-to-vacuum transition amplitude may be explicitly derived in functional *differential* form, in a unified manner, leading to modifications of computational rules by including such factors as Faddeev–Popov ones [14, 15] and *modifications* thereof. The derivation is given in the *presence* of external sources, without recourse to path integrals, and without relying on any symmetry and invariance arguments. There has also been a renewed interest in Schwinger's action principle recently (see, e.g., [16–18]) emphasizing, in general, however, operator aspects of a theory, as deriving, for example, commutation relations, rather than dealing with computational ones related directly to generating functionals as done here.

2 General Class of Lagrangians

Consider Lagrangian densities which may depend on one or more coupling constants. We scale these couplings by a parameter λ which is eventually set equal to one. The resulting Lagrangian densities will be denoted by $\underline{\mathcal{L}}(x;\lambda)$. The class of Lagrangian densities considered are of the following types

$$\underline{\mathcal{L}}(x;\lambda) = \mathcal{L}(x;0) + \mathcal{L}_I(x;\lambda) + J_1^i(x)\chi_i(x) + J_2^j(x)\eta_j(x) \tag{1}$$

where $\chi_i(x)$ and $\eta_j(x)$ are independent and dependent fields, respectively. $J_1^i(x)$, $J_2^j(x)$ are external sources coupled to these respective fields. The interaction Lagrangian densities sought are of the following forms

$$\mathcal{L}_I(x;\lambda) = B(x;\lambda) + B^j(x;\lambda)\eta_j(x) + \frac{1}{2}B^{jk}(x;\lambda)\eta_j(x)\eta_k(x) \tag{2}$$

with $\mathcal{L}_I(x;0) = 0$, where $\partial B(x;\lambda)/\partial\lambda$, $\partial B^j(x;\lambda)/\partial\lambda$, $\partial B^{jk}(x;\lambda)/\partial\lambda = \partial B^{kj}(x;\lambda)/\partial\lambda$ may be expressed in terms of the independent fields, and the latter two may involve space derivatives applied to the dependent fields $\eta_j(x)$. By definition, the canonical conjugate momenta of the fields $\eta_j(x)$ vanish. That is, formally, $\partial\underline{\mathcal{L}}(x;\lambda)/\partial(\partial_0\eta_j(x)) = 0$. Let $\partial\mathcal{L}(x;0)/\partial\eta_j(x) = A^{jk}(x)\eta_k(x)$. The *constraint* equation of the dependent fields $\eta_k(x)$ follow from (1, 2) to be

$$M^{jk}(x;\lambda)\eta_k(x) = -[B^j(x;\lambda) + J_2^j(x)] \tag{3}$$

where

$$M^{jk}(x;\lambda) = A^{jk}(x) + B^{jk}(x;\lambda) \tag{4}$$

Let $D_{jk}(x,x';\lambda)$ denote the Green operator function satisfying

$$M^{ij}(x;\lambda)D_{jk}(x,x';\lambda) = \delta^i{}_k\delta^4(x,x') \tag{5}$$

From (3), this leads to

$$\eta_j(x) = -\int(\mathrm{d}x')D_{jk}(x,x';\lambda)[B^k(x';\lambda) + J_2^k(x')] \tag{6}$$

giving a constraint which is *explicit* source J_2^k-dependent, and is also a function of the independent fields.

Let $|0_{\mp}\rangle$ denote the vacuum states of a theory before/after the external are switched on/off, respectively. We are interested in the variation of the vacuum-to-vacuum transition amplitude $\langle 0_+|0_-\rangle$, governed by the Lagrangian density $\underline{\mathcal{L}}(x;\lambda)$ in (1), with respect to the parameter λ as well as the external sources $J_1^i(x)$, $J_2^j(x)$. To this end, we invoke the quantum dynamical principle which states (see, e.g., [7–10, 12, 13])

$$\frac{\partial}{\partial\lambda}\langle 0_+|0_-\rangle = \mathrm{i}\,\langle 0_+|\int(\mathrm{d}x)\frac{\partial}{\partial\lambda}\mathcal{L}_I(x;\lambda)\,|0_-\rangle \tag{7}$$

$$(-\mathrm{i})\frac{\delta}{\delta J_1^i(x)}\langle 0_+|0_-\rangle = \langle 0_+|\,\chi_i(x)\,|0_-\rangle \tag{8}$$

$$(-\mathrm{i})\frac{\delta}{\delta J_2^j(x)}\langle 0_+|0_-\rangle = \langle 0_+|\,\eta_j(x)\,|0_-\rangle \tag{9}$$

Consider the matrix element $\langle 0_+|\,F(x;\lambda,J_1,J_2)\,|0_-\rangle$ of an operator which is not only a function of the independent fields but which may also have an explicit dependence on λ and the external sources J_1^i, J_2^j. An explicit λ, J_2^j dependence may occur, for example, when the dependent fields $\eta_j(x)$ are expressed in terms of the independent fields and J_2^j as given in (6).

The quantum dynamical principle, in particular, then states (see [11–13]) that

$$\begin{aligned}(-\mathrm{i})&\frac{\delta}{\delta J_2^j(x')}\langle 0_+|\,F(x;\lambda,J_1,J_2)\,|0_-\rangle\\ &= \langle 0_+|\,(F(x;\lambda,J_1,J_2)\eta_j(x'))_+\,|0_-\rangle - \mathrm{i}\,\langle 0_+|\,\frac{\delta}{\delta J_2^j(x')}F(x;\lambda,J_1,J_2)\,|0_-\rangle\end{aligned} \tag{10}$$

where $(\ldots)_+$ denotes the time-ordered product, and the functional derivative, with respect to $J_2^j(x')$, in the second term on the right-hand side of (10), is applied to the explicit J_2-dependent term (if any) that occurs in F.

Let $\partial B'^j(x;\lambda)/\partial\lambda$ denote $\partial B^j(x;\lambda)/\partial\lambda$ with the fields $\chi_i(x)$ in the latter replaced by the functional derivatives $(-\mathrm{i})\delta/\delta J^i(x)$. From (8–10), we then have

$$(-\mathrm{i})\frac{\delta}{\delta J_2^k(x')}\frac{\partial}{\partial\lambda}B'^j(x;\lambda)\,\langle 0_+|0_-\rangle = \langle 0_+|\left(\frac{\partial}{\partial\lambda}B^j(x;\lambda)\eta_k(x')\right)_+|0_-\rangle \tag{11}$$

where we have used the fact that $\partial B^j(x;\lambda)/\partial\lambda$ is expressed in terms of the independent fields and has no explicit J_2^k-dependence, and hence the second term on the right-hand side of (10) is zero for this corresponding case.

On the other hand, (10) also gives

$$\begin{aligned}(-\mathrm{i})&\frac{\delta}{\delta J_2^j(x'')}(-\mathrm{i})\frac{\delta}{\delta J_2^k(x')}\frac{\partial}{\partial\lambda}B'^{jk}(x;\lambda)\,\langle 0_+|0_-\rangle\\ &= \langle 0_+|\left(\frac{\partial}{\partial\lambda}B^{jk}(x;\lambda)\eta_j(x'')\eta_k(x')\right)_+|0_-\rangle - \mathrm{i}\,\langle 0_+|\left(\frac{\partial}{\partial\lambda}B^{jk}(x;\lambda)\frac{\delta}{\delta J_2^j(x'')}\eta_k(x')\right)_+|0_-\rangle\end{aligned} \tag{12}$$

where from (6),

$$\frac{\delta}{\delta J_2^j(x'')}\eta_k(x') = -D_{kj}(x',x'';\lambda) \tag{13}$$

Hence the second term on the right-hand side of (12) is simply

$$+\mathrm{i}\frac{\partial}{\partial\lambda}B'^{jk}(x;\lambda)\ D'_{kj}(x',x'';\lambda)\,\langle 0_+|0_-\rangle \tag{14}$$

with $D'_{kj}(x',x'';\lambda)$ denoting $D_{kj}(x',x'';\lambda)$ with the fields $\chi_i(x)$ replaced by $(-\mathrm{i})\delta/\delta J_1^i(x)$.

All told, we may *solve* for $\langle 0_+|\,\partial\mathcal{L}_I(x;\lambda)/\partial\lambda\,|0_-\rangle$ in terms of functional derivatives, with respect to the external sources, as *applied to* $\langle 0_+|0_-\rangle$ directly from (2, 7, 11–14) to obtain

$$\frac{\partial}{\partial\lambda}\langle 0_+|0_-\rangle = \Bigg[\mathrm{i}\int(\mathrm{d}x)\frac{\partial}{\partial\lambda}\mathcal{L}'(x;\lambda) + \frac{1}{2}\int(\mathrm{d}x)\bigg(\frac{\partial}{\partial\lambda}B'^{jk}(x;\lambda)\bigg)\ D'_{kj}(x,x;\lambda)\Bigg]\langle 0_+|0_-\rangle \tag{15}$$

where $\mathcal{L}'_I(x;\lambda)$ denotes $\mathcal{L}_I(x;\lambda)$ with $\chi_i(x)$, $\eta_j(x)$ replaced in the latter by $(-\mathrm{i})\delta/\delta J_1^i(x)$, $(-\mathrm{i})\delta/\delta J_2^j(x)$, respectively.

Upon integrating (15) over λ from 0 to 1, gives

$$\langle 0_+|0_-\rangle = \exp\bigg(\mathrm{i}\int(\mathrm{d}x)\mathcal{L}'_I(x)\bigg) \times\exp\Bigg[\frac{1}{2}\int(\mathrm{d}x)\int_0^1\mathrm{d}\lambda\,\bigg(\frac{\partial}{\partial\lambda}B'^{jk}(x;\lambda)\bigg)\ D'_{kj}(x,x;\lambda)\Bigg]\langle 0_+|0_-\rangle_0 \tag{16}$$

where $\langle 0_+|0_-\rangle_0$ is governed by the Lagrangian density $[\mathcal{L}(x;0)+J_1^i(x)\chi_i(x)+J_2^j(x)\eta_j(x)]$ in (1), and $\mathcal{L}'_I(x)\equiv\mathcal{L}'_I(x;0)$, with the latter defined below (15).

Equation (16) provides the solution for the generating functional $\langle 0_+|0_-\rangle$ in the *presence* of external sources. We thus see that *for interaction Lagrangian densities such that* $\partial\mathcal{L}_I(x;\lambda)/\partial\lambda$ *are quadratic in dependent fields* ($\partial B^{jk}(x;\lambda)/\partial\lambda\neq 0$), *as described above, the rules for computations, via the generating functional* $\langle 0_+|0_-\rangle$ *are modified by the presence of the multiplicative functional differential operator factor*

$$\exp\Bigg[\frac{1}{2}\int(\mathrm{d}x)\int_0^1\mathrm{d}\lambda\bigg(\frac{\partial}{\partial\lambda}B'^{jk}(x;\lambda)\bigg)D'_{kj}(x,x;\lambda)\Bigg] \tag{17}$$

As special cases of the general Lagrangians described through (1) and developed above, consider *non-abelian* gauge theories with Lagrangian densities

$$\underline{\mathcal{L}} = \mathcal{L} + \mathcal{L}_S \tag{18}$$

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} + \frac{1}{2\mathrm{i}}\big[\partial_\mu\bar{\psi}\gamma^\mu\psi - \bar{\psi}\gamma^\mu\partial_\mu\psi\big] - m_o\bar{\psi}\psi + g_o\bar{\psi}\gamma_\mu A^\mu\psi \tag{19}$$

$$\mathcal{L}_S = \bar{\rho}\psi + \bar{\psi}\rho + J_a^\mu A^a_\mu \tag{20}$$

$$A_\mu = A^a_\mu t_a,\qquad G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g_o[A_\mu, A_\nu] \tag{21}$$

$$G_{\mu\nu} = G^a_{\mu\nu}t_a \tag{22}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_o f^{abc}A^b_\mu A^c_\nu \tag{23}$$

The t^a matrices are generators of the underlying algebra, and the f^{abc}, totally anti-symmetric, are the structure constants satisfying the Jacobi identity $[t^a, t^b] = \mathrm{i} f^{abc} t^c$. $\mathcal{L}_S$ is the source term with the J_a^μ classical functions, while ρ, $\bar{\rho}$ are so-called anti-commuting Grassmann variables.

Upon setting

$$\nabla_\mu^{ab} = \delta^{ab}\partial_\mu + g_o f^{acb} A_\mu^c \tag{24}$$

working in the Coulomb gauge $\partial_i A_a^i = 0$, $i = 1, 2, 3$, and introducing the Green operator function $D^{cd}(x, x'; g_o)$, satisfying

$$[\delta^{ac}\boldsymbol{\partial}^2 + g_o f^{abc} A_k^b \partial_k] D^{cd}(x, x'; g_o) = \delta^4(x, x')\delta^{ad} \tag{25}$$

$k = 1, 2, 3$, one may solve for G_a^{k0} (see [11]) as follows

$$G_a^{k0} = -\partial^k D_{ab} J_b^0 + F_a^k \tag{26}$$

in a matrix notation, where F_a^k is *not an explicit* function of the dependent fields and of the external sources. From the very definition of G_a^{k0} in (23), we also have

$$\begin{aligned}\partial_k G_a^{k0} &= \nabla_k^{ab}\partial_k A_b^0 \\ &= [\delta^{ab}\boldsymbol{\partial}^2 + g_o f^{acb} A_k^c \partial_k] A_b^0\end{aligned} \tag{27}$$

Hence we may solve for A_b^0 to obtain

$$A_b^0 = -D_{bc}\boldsymbol{\partial}^2 D_{ca} J_a^0 + K_b \tag{28}$$

where K_b is not an explicit function of the external sources.

Now we show that the time derivative $\partial_0 A_b^k$ may be solved in terms of A_c^0 and the independent fields themselves. To this end, we note that

$$\partial^0 A_a^k = \nabla_{ab}^k A_b^0 - G_a^{k0} \tag{29}$$

and from (28)

$$D_{ca} J_a^0 = -\left[\delta_{ca} + \frac{1}{\boldsymbol{\partial}^2} f^{cda} A_k^d \partial_k\right](A_a^0 - K_a) \tag{30}$$

Accordingly, (29, 30) and (26) lead to

$$\partial^0 A_a^k = g_o f^{acb}\left[A_c^k \frac{\partial^l}{\boldsymbol{\partial}^2} - \frac{\partial^k}{\boldsymbol{\partial}^2} A_c^l\right]\partial_l A_b^0 + L_a^k \tag{31}$$

where L_a^k has no explicit dependence on the external sources and on the dependent fields A_b^0.

Finally we note that

$$\frac{\partial}{\partial g_o}\mathcal{L}_I = -f^{abc} A_k^b\left(A_0^c G_a^{k0} + \frac{1}{2} A_l^c G_a^{kl}\right) + \bar{\psi}\gamma^\mu A_\mu \psi \tag{32}$$

From the definition of G_a^{k0} in (23), *and the fact that* $\partial^0 A_a^k$ may be expressed in terms of the A_b^0, as shown in (31), and the independent fields themselves, we see that (32) is *quadratic* in the dependent fields A_b^0.

The structure G_a^{k0} in (23) may be expressed, from (31), as a linear function of the dependent fields A_a^0, and directly from (26) we have

$$\frac{\delta}{\delta J_b^\nu} G_a^{k0}(x) = -\delta^0{}_\nu\, \partial_k D_{ab}(x, x'; g_o) \tag{33}$$

Hence (7–12, 32, 33) give

$$\begin{aligned}\frac{\partial}{\partial g_o} \langle 0_+ | 0_- \rangle = \Bigg[\mathrm{i} \int (\mathrm{d}x) \frac{\partial}{\partial g_o} \mathcal{L}'_I(x; g_o) \\ - \int (\mathrm{d}x) f^{bca} A_k^b(x) D'^{ac}(x, x; g_o) \Bigg] \langle 0_+ | 0_- \rangle\end{aligned} \tag{34}$$

where $A_k'^b(x) = (-\mathrm{i})\delta/\delta J_b^k(x)$. Upon using the definition of $D^{ac}(x, x', g_o)$ in (25), and integrating (34) over g_o, we obtain the modifying Faddeev–Popov multiplicative factor

$$\exp \mathrm{Tr} \ln \left[1 - \mathrm{i} g_o \frac{1}{\boldsymbol{\partial}^2} A'_k \partial^k \right] \tag{35}$$

as a *special case* of (17), where

$$\mathrm{Tr}[f] \equiv \int (\mathrm{d}x)\, f^{aa}(x, x) \tag{36}$$

The general derivation given above for interaction Lagrangian densities such that $\partial \mathcal{L}_I(x; \lambda)/\partial\lambda$ may be expressed as quadratic functions in dependent fields involves no symmetry arguments. As a matter of fact, we may consider the addition of a gauge-invariant breaking term $(g_1/2) A_a^\mu A_\mu^a \bar{\psi}\psi$ to the Lagrangian density in (19) which is again quadratic in A_a^0 and presumably contributes to the generation of masses to the vector fields through a non-vanishing expectation value of $\bar{\psi}\psi$. A detailed analysis shows (see [11]) that the modifying *extra multiplicative factor* to $\exp(\mathrm{i}\int (\mathrm{d}x) \mathcal{L}'_I(x)) \langle 0_+ | 0_- \rangle |_0$ occurring in $\langle 0_+ | 0_- \rangle$ is given by

$$\begin{aligned}\exp\left[-\frac{1}{2} \mathrm{Tr} \ln \left(1 + \frac{g_1}{\nabla'_l \partial_l (\boldsymbol{\partial}^2)^{-1} \nabla'_k \partial_k} \bar{\psi}' \psi' \right) \right] \\ \times \exp \mathrm{Tr} \ln \left(1 - \mathrm{i} g_o \frac{1}{\boldsymbol{\partial}^2} A'_k \partial_k \right)\end{aligned} \tag{37}$$

where $\bar{\psi}' = (-\mathrm{i})\delta/\delta\rho$, $\psi' = (-\mathrm{i})\delta/\delta\bar{\rho}$, and $\mathcal{L}'_I$ is the *new* interaction Lagrangian density functional differential operator expressed in terms of functional derivatives with respect to the external sources.

3 Conclusion

We have seen, within the functional differential formalism of quantum field theory in the presence of external sources, that interaction Lagrangian densities $\mathcal{L}_I(x; \lambda)$ such that $\partial \mathcal{L}_I(x; \lambda)/\partial\lambda$ may be expressed as quadratic functions of dependent fields (i.e., $\partial B^{jk}(x; \lambda)/\partial\lambda \neq 0$ in (2)) and *arbitrary* functions of independent fields, necessarily lead to modifications of the rules for computations, via the generating functional $\langle 0_+|0_-\rangle$ as a functional of the external sources which are coupled to the fields, and no appeal was made, through the analysis, to path integrals. The general expression for such a modification is given in (17) as a functional differential operator occurring as a multiplicative factor in $\langle 0_+|0_-\rangle$. Such Lagrangians play *central* roles in fundamental physics and present renormalizable gauge theories fall into this category. It is important, however, to emphasize that such modifications are not tied up to non-abelian gauge theories, through the emergence of so-called Faddeev–Popov factors, as one might naively expect, but apply to theories which, in general, are quadratic functions in dependent fields as described above. As a matter of fact the addition of a gauge term breaking term in the form $(g_1/2)A^\mu A_\mu \bar{\psi}\psi$ to the interaction Lagrangian density of QED (abelian gauge theory), which is again quadratic in A^0, leads, according to (37), the following extra functional differential multiplicative factor

$$\exp\left[-\frac{1}{2}\,\mathrm{Tr}\ln\left(1+\frac{g_1}{\boldsymbol{\partial}^2}\bar{\psi}'\psi'\right)\right] \tag{38}$$

multiplying $\exp[\mathrm{i}\int(\mathrm{d}x)\mathcal{L}_I'(x)]\,\langle 0_+|0_-\rangle_0$, where $\mathcal{L}_I'(x)$ is the *new* interaction Lagrangian density functional differential operator including the additional term just mentioned as a simplified version of (37). That is, *a non-trivial modification arises even for such an abelian gauge theory*. The technical question now arises as to what happens to model Lagrangian densities that one may set up which are cubic or of higher order in dependent fields in the sense investigated above. The main complication with such theories becomes obvious by noting that the corresponding Green function operator function to the one in (5) will now *depend* on dependent fields themselves. Accordingly, when we apply the corresponding rule in (12) for finally expressing the matrix element $\langle 0_+|\,(\partial\mathcal{L}_I/\partial\lambda)\,|0_-\rangle$, as a functional differential operator, with respect to the external sources, to be eventually applied to $\langle 0_+|0_-\rangle$, the expression $\delta\eta_k(x')/\delta J_2^j(x'')$ will again depend, rather non-trivially, on the dependent fields $\eta_j(x)$. This makes the procedure of expressing the matrix element just mentioned as a functional differential operation to be applied to $\langle 0_+|0_-\rangle$ quite unmanageable. Such field theories require very special tools and will be investigated, within the functional differential formalism, in a subsequent report.

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